Yielding of Commercial Poly(vinyl chloride) Pipe Material

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ABSTRACT: Tubular test specimens of commercial poly(vinyl chloride) pipe material were tested in biaxial stress systems covering the four quadrants of the plane–stress space. The test rig provided yield stresses at a constant temperature and controlled strain rate tension, compression, internal pressure, external pressure, and torsion combined loading. The experimental yield points fit the von Mises yield criterion. This result was confirmed by the ratio of compressive to tensile yield stresses; being unity, the density remaining constant with the applied hydrostatic stress, and, finally, the normality of the strain vector to the yield locus at the corresponding stressing point. Subsequent yielding loci of previously deformed specimens deviated largely from the von Mises locus. This deviation may have been caused by craze–void interaction. Finally, the viscous behavior, the dependence of yield stresses on the strain rate and temperature, is represented with a generalized phenomenological model based on an Eyring–Ward-type expression. © 2001 John Wiley & Sons, Inc. J Appl Polym Sci 81: 991–999, 2001

Key words: poly(vinyl chloride); piping material; yield criterion; constitutive law; temperature and strain-rate effects; postyield behavior

INTRODUCTION

Design principles for elements made of polymeric materials consider mainly elastic, plastic, and time-dependent deformations. In the case of excessive short-time loading, yielding and inelastic or plastic deformation seem to be the dominating failure phenomena. Postyielding plastic deformation is usually accompanied by crazes and stress whitening, indicating microscale cracklike and void development.

Several yield criteria have been proposed to describe the behavior of different polymers, as reviewed by Ward.¹ Also, a direct comparison between measured data and alternate yield criteria for specific polymers has been presented in the literature.

Whitney and Andrews² studied the behavior of polystyrene (PS) and other polymeric materials with particular attention to the effect of the hydrostatic component of stress on the yield behavior and volume changes occurring before and during yielding. They tested specimens of different geometries, depending on the type of loading combination. Their results, which unjustifiably neglected the effect of strain rate, showed that neither Tresca nor von Mises criteria, as applied to metals, would be a suitable description for yielding, but instead a modified Tresca criterion in the form $\tau = \tau_c - \alpha \sigma_m$, would be more appropriate. In a more detailed study, Bowden and Jukes³ examined the biaxial yield behavior of poly(methyl methacrylate) (PMMA) as initially presented by Williams and Ford⁴ at room temperature. Their results fit the modified Tresca criterion.

Rabinowitz et al.,⁵ among others,^{6–8} proposed a modified von Mises criterion for yielding in the form $\tau_{oct} = \tau_o - \mu \sigma_m$. The coefficient $\mu = 0.11$ for

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poly(vinyl chloride) (PVC)⁶ resulted in elliptical yield locus-fitting tension-torsion experiments. Thorkildsen,⁹ however, quoted some experiments on the yielding behavior of thin-walled tubes of PMMA without quoting the strain-rate effects and adopted a yield stress definition at 0.2% strain; the results were shown to fit the von Mises criterion.

Raghava et al.¹⁰ investigated the yield behavior of several polymers, including PVC, at room temperature. Their results were presented in the tension-tension and compression-compression quadrants of the plane-stress space. The ratio $(\sigma_{YC}/\sigma_{YT})$ between the compressive and tensile yield strengths, around 1.3 for PVC, was successfully accommodated with a pressure-modified von Mises criterion. Raghava et al. recommended further tests in the third quadrant of the planestress space.

In a more recent study, Quinson et al.¹¹ tested polycarbonate (PC), PS, and PMMA at different temperatures in simple tension, compression, and shear as well as plane strain compression. Using a different specimen geometry for each test, they found that a modified Tresca criterion was more appropriate for describing shear band deformation. A modified von Mises criterion fits better homogeneous deformation. In their experiments, they determined the yield stress point via residual strain measurement after unloading.

Tuttle et al.¹² experimentally investigated the elastic and yield behavior of mildly and transversely isotropic high-density polyethylene (HDPE) tubes in the first quadrant of the stress space. The data of yielding fit reasonably well a pressure-modified Hill anisotropic criterion previously suggested by Caddell et al.¹³ with the assumption $\sigma_{YC}/\sigma_{YT} = 1.12$.

In actual practice, commercially used polymeric blends are composed of pure resin and additives (e.g., fillers, colors, and stabilizers). It is known that the presence of these additives affects the behavior of this compound at yielding. Insoluble additives in PVC blends decrease its yield strength, whereas soluble ones affect the behavior oppositely, rendering it more brittle.¹⁴ The decrease of the tensile yield stress of PVC due to the increasing volume concentration of calcium carbonate as a filler has been explained by the weak coupling between the base polymer and the filler.¹⁵ In this study, the yielding behavior of commercially available PVC piping material was investigated with unified test-specimen geometry. Full-loading combinations covering all sectors of



Figure 1 Tubular test specimen.

the plane-stress space were considered. Special attention was given to the application of a unified definition for the yield point determined under the same conditions of strain rate and temperature.

EXPERIMENTAL

Material and Specimens

Commercially rigid PVC pipes were used to prepare test specimens. These pipes were 60 mm in diameter and 5.3 mm thick with sustainable working pressure of 1 MPa. The glass-transition temperature was 81°C. The composition of the compound used to manufacture the rigid PVC pipes contained additives such as calcium powder, carbon black, and anitburning agent. Tubular test specimens (Fig. 1) were prepared from the PVC pipes by careful, well-cooled machining up to a thickness of 2 mm over a gauge length of 30 mm. Tensile test specimens, according to ASTM D 638-86,¹⁶ were prepared by longitudinal slitting from the PVC pipes.

Experimental Setup

The experimental setup (Figs. 2 and 3) was designed and constructed to conduct experiments on tubular PVC specimens under combined loading conditions. Loading included tension, compression, torque, and internal and external pressurization. All loads were applied with hydraulic means except for torque, which was applied through dead weights and a pulley system. Arrangements for elevated temperatures up to 100 \pm 1.0°C were provided with an appropriate oven. Variable loading rates were made possible by the



Figure 2 Layout of the testing system.

controlled flow rate of a positive displacement pump furnishing the required pressure.

Tensile and compressive loads were measured with strain gauge-type load cells, whereas pressure was monitored with a piezoelectric transducer. Linear and angular displacements were measured with a linear voltage displacement transducer and an appropriate potentiometer, respectively. Plots of the various load combinations versus displacements were recorded.

Preliminary Tests

Preliminary tests on the PVC material under consideration led to the adoption of a unified definition for the yield point throughout the test program. Either the occurrence of a maximum point or a plateau in the applied load⁵ defined the yield point. In essence, this was similar to metals, for which the point of maximum load defines the onset of nonhomogeneous flow. This definition also conforms to the ASTM D 638-86¹⁶ definition of a yield point of a polymer being the first point on the σ - ϵ curve at which an increase in the strain occurs without an increase in stress. There was no need to use the offset yield strength as adopted by others (e.g., refs. 9, 10, and 12).

To ensure the validity of the selected tubular specimens, tensile tests were conducted on standard specimens to characterize the PVC material under consideration at various strain rates. Figure 4 shows the variation of the yield stress versus the strain rate for a standard tensile specimen and a tubular specimen. The results of both types of specimens were almost coincident.

Further testing on tubular specimens was conducted with the specimens subjected to tensile stress in either the axial direction or the hoop direction (i.e., a pressurized open tube). The results indicated the isotropic behavior of the tested PVC material. Moreover, uniaxial tests with the tubular specimen geometry on the same test rig



Figure 3 Test rig details.

gave a ratio of unity between the tensile and compressive stresses.

To obtain a relation between the yield stress, strain-rate, and temperature, experiments on tubular specimens comprising axial and hoop tension, compression, and proportional combined internal pressure and tension were conducted. Test results were obtained at various strain rates,



Figure 4 Tensile test results of tubular test specimens compared to those of the standard tensile test specimen.

ranging from 0.67×10^3 to 12.6×10^{-3} per second, and at temperatures of 18, 32, and 75°C. The effective stresses at yielding versus the strain rates at these temperatures are shown in Figure 5.

RESULTS AND DISCUSSION

Tracing the Yield Loci

Initial Yield Locus

As mentioned previously, the yield point was taken to be corresponding to the peak stress read from the measured load-displacement plots. Experimental conditions of the temperature and strain rate were controlled as much as the equipment permitted. Often, this control was not easy to achieve under conditions of combined loading. Hence, a relation fitted to the results of Figure 5 corrects results of yielding for the effects of slight deviations from the required constant strain rate.

Figures 6 and 7 show the yield loci at various strain rates and temperatures, respectively. It is clear that the tested PVC material followed the von Mises yield function in all stress quadrants. The yield locus expanded uniformly as the strain rate increased and the temperature decreased.

A check of the conformity of the PVC material to von Mises criterion was achieved by the observation of the normality of the strain vector to the



Figure 5 Effect of the strain rate and temperature on the yield stress as determined at different loadings. The fitted lines represent eq. (7).

yield surface. Discrepancies between the direction of the measured strain vector at yielding and the normal to the ellipse did not exceed 0.01 rad within the first stress quadrant of the yield locus



Figure 6 Comparison of von Mises yield loci with experimental results at 18°C for different strain rates.



Figure 7 Comparison of experimental yield points with von Mises yield loci as predicted from eq. (7) at different temperatures and strain rates.

corresponding to $\epsilon = 8.4 \times 10^{-3} \text{ s}^{-1}$ and 18°C. Another verification was realized by checking that the volume of the tested PVC material at yielding remained constant. This condition was accurately satisfied by measurements of density being weighed within $\pm 10^{-4}$ g as shown in Figure 8. This was further confirmed by the plotting of the yield stress against the hydrostatic stress component. As shown in Figure 9, the yield stress, as expected for shear yielding, was independent of the mean stress for the three testing temperatures.

The fact that the behavior of the tested tube PVC material obeyed the von Mises criterion was confirmed by the fact that the ratio between the tensile and compressive yield stresses was unity, as mentioned previously. A general pressure-modified von Mises yield condition (notably different from the modified von Mises one, $\tau_{oct} = \tau_o + \mu \sigma_m$) was first applied by Raghava et al.¹⁰ It takes the form

$$\begin{aligned} A(\sigma_1 + \sigma_2 + \sigma_3) + B[(\sigma_1 - \sigma_2)^2 \\ &+ (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] = 1 \end{aligned} (1)$$

where A and B are material parameters. Reducing this to the cases of uniaxial tension and compression gives



Figure 8 Measured density changes at different mean stress components.

$$A = (\sigma_{YC} - \sigma_{YT}) / (\sigma_{YC} \sigma_{YT})$$

and
$$B = 1 / (2\sigma_{YC} \sigma_{YT}) \quad (2)$$

For $\sigma_{YC} = \sigma_{YT} = \sigma_Y$, A = 0 and $B = 1/(2\sigma_Y^{2})$, and the yield condition, eq. (1) reduces to von Mises. However, the sensitivity of the yield behavior to $\sigma_{YC} / \sigma_{YT}$ must be taken into consideration. For instance, by assuming a value of 1.1 for this ratio, eq. (1) indicates errors in predicting yielding by the von Mises criterion up to -4.6 and +15.3% for the states of equibiaxial tension and equibiaxial compression, respectively. Hence, for safe design purposes, this would imply testing the polymer blend in its commercial form to describe its yield behavior rather than relying on the data obtained on the base polymer.

The conformity of the measured yield behavior of this PVC with the von Mises criterion has been reported for other polymeric materials. Ewing et al.¹⁷ indicated that the behavior of polythene tubes under combined stresses is essentially similar to that under tension, and the concept of effective stress and effective strain is applicable within accuracies for engineering applications. Recently, Quinson et al.¹¹ found a rather weak dependence of the yield stress of PC on pressure by finding a coefficient μ of 0.03–0.05 at 20°C with the modified von Mises criterion. Also, Raghava et al.¹⁸ demonstrated that the true stress–true strain behaviors for PC, PVC, and acrylonitrile-butadiene-styrene (ABS)/PVC laminate subjected to uniaxial tension and equibiaxial bulging are similar. Such agreement was not, however, observed for HDPE. They argued that the good agreement observed for these polymers suggests that uniaxial tensile and compressive behaviors are similar. Note that bulging experiments produced equivalent compressive stresses unaffected by friction; in this, they differ from conventional compression tests, which have platen friction. This argument, however, lacks quantitative evaluation because, in careful lubricated compression testing, the low coefficient friction results in only a slight increase in the compressive load.

Subsequent Yield Locus

The conformity to the von Mises criterion was not observed for PVC specimens that were deformed beyond yielding up to relatively large strains, for example, $\epsilon = 0.1$. These specimens did not obey the volume constancy condition, as shown in Figure 9. The lowest density was recorded at a mean stress of 31.6 MPa for equibiaxial tension. The change in density was smaller with decreasing mean stress, where it attained a value of 1.41 g/cm³ at compressive mean stresses up to -27MPa in biaxial compression. This density change may have been due to void nucleation and growth associated with craze formation.¹⁹ Such an obser-



Figure 9 Effect of the hydrostatic stress component on the yield stress at different temperatures.



Figure 10 Subsequent yield loci at a strain rate of 8.4 $\times 10^{-3} \text{ s}^{-1}$ and 18°C. The solid line represents a von Mises ellipse, whereas the broken line represents calculations from eq. (3) together with a power hardening law.

vation suggested tracing subsequent yield loci for the tested PVC material, as shown in Figure 10. The second and third yield loci plotted in this figure were obtained for previously postyielded specimens. The results indicated more flattening in the yield locus (i.e., decreasing yield stress at biaxial stress states and progressively increasing yield stresses toward a uniaxial tensile state of stress).

An explanation for these observed subsequent vield loci may stem from the formation of a craze microstructure that is initiated when an applied tensile stress below the bulk yield causes microvoids to nucleate at points of high stress concentration in the polymer. As suggested by Argon,¹⁹ when the local porosity reaches a critical level, visible craze nuclei form, being governed by the rate of the plastic expansion of microvoids. Crazes are thus microstructural regions of highly plastically deformed material interspersed with voids (i.e., regions of a set of microfibrils elongated in the direction of maximum tensile stress with a large intensity of voids). At the subsequent deformation up to yielding, two counteracting phenomenological behaviors may be present. The first is the strain hardening that arises from polymer chains in the fibrils undergoing molecular orientation, thus enhancing the strength and stability of the craze structure.^{20,21} The second is softening due to plastic cavitation and void interaction that is dependent on the state of hydrostatic tension, as seen from the Gurson–Tvergaard flow equation:²²

$$\bar{\sigma}_Y^2 = \bar{\sigma}_M^2 (1 + q^2 C_v^2) - 2C_v q \bar{\sigma}_M^2 \cosh(3q \sigma_m/2\bar{\sigma}_M) \quad (3)$$

In this equation, $\bar{\sigma}_M$ is the flow stress expressed by the first yield criterion, whereas C_v is the void ratio as measured by density changes at different hydrostatic stress levels in Figure 9. Equation (3) may be used, therefore, to predict the amount of softening due to voids present at subsequent vielding. Such softening is either overcompensated by the amount of strain hardening of oriented craze fibrils, thus resulting in the enlargement of the subsequent yield loci, as shown in Figure 10, close to uniaxial loading regions. In other regions where higher stress biaxiality prevails and void growth is favored,²⁰ softening overwhelms hardening, thus reducing subsequent yield stresses. Sample calculations with strain hardening coefficient of 0.16 for the PVC material result in a locus, shown in Figure 10 with a dotted line, that agrees at least qualitatively with the experimentally determined second subsequent yield locus. Such agreement cannot be considered conclusive, and further observations for the microstructural events during the deformation of this particular PVC compound are needed.

Viscous Nature of Yield Stress

The yield behavior of PVC is seen to be very dependent on the temperature and strain rate. Many investigators (e.g., Ward et al.²³) have employed Eyring's theory of viscosity²⁴ to model this behavior, including the effect of the hydrostatic component of stress:

$$\dot{\varepsilon} = \Gamma \exp[-(\Delta E^* - \sigma_Y v - \sigma_m \Omega)/(RT)] \quad (4)$$

Recently, Lesser and Kody²⁵ suggested, on the basis of the Eyring–Ward models, an equation expressing the dependence of yielding on the strain rate and temperature:

$$\bar{\sigma}_{Y} = \sigma_{Y_{o}}(\dot{\varepsilon}_{r}, T_{g}) + (R/v)T_{g}\ln(\dot{\varepsilon}/\dot{\varepsilon}_{r}) + (R/v)(T - T_{g})\ln(\dot{\varepsilon}/\Gamma) - \Omega\sigma_{m}$$
(5)

where

$$\begin{split} \bar{\sigma}_Y &= (1/\sqrt{2}) [(\sigma_1 - \sigma_2)^2 \\ &+ (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \end{split}$$

and

$$\dot{\tilde{\varepsilon}} = (\sqrt{2}/3) [(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_2 - \dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_3 - \dot{\varepsilon}_1)^2]^{1/2}$$

In this expression, σ_{Yo} is the yield stress determined at the glass-transition temperature, T_g , and some reference shear strain rate, $\dot{\varepsilon}_r$. The constant, R, is the gas constant, and v is the activation volume for shear flow. Lesser and Kody²⁵ applied this model to epoxy networks subjected to biaxial stress systems. They found that eq. (5) offers a generalized model capable of predicting the yield stress at a given strain rate and temperature for any general state of stress. Equation (5) has the advantage of being a "working model"²⁵ in the sense that the parameters used in it can be readily determined from standard mechanical and thermal conventional tests.

To apply this model to this experimental data on PVC piping material, two plots of σ_Y/T versus the strain rate ($\dot{\varepsilon}$) and σ_Y versus $T - T_g$ were generated with the data from Figure 5. The parameter $\bar{\sigma}_{Yo}$ at $\dot{\varepsilon}$ and T_g was then determined (as being 17.28 MPa at $T_g = 81^{\circ}$ C and $\dot{\varepsilon} = 0.0004$ s⁻¹). The ratio (R/v) was obtained with the data from Figure 5 representing the yield stress versus strain rate, corresponding to 18°C, by fitting the relationship

$$R/v = (\sigma_{Y2} - \sigma_{Y1})/(T \ln \dot{\varepsilon}_2/\dot{\varepsilon}_1)$$
(6)

where σ_{Y2} and σ_{Y1} are the yield stresses measured at two different strain rates, $\dot{\varepsilon}_2$ and $\dot{\varepsilon}_1$, respectively. The constant Γ is thus determined with any data point from Figure 5 for 18°C. The previous fitting procedure assumes a constant T_g regardless of the observation that it depends on the hydrostatic stress value as reported by O'Reilly, according to $\partial T_g/\partial \sigma_m = 0.016$ for PVC.²⁶ The resulting expression in terms of $\bar{\sigma}_Y$ (MPa), T (K), and $\dot{\varepsilon}$ (s⁻¹)—recalling that in this case $\Omega = 0$ —is thus

$$\bar{\sigma}_Y = 17.28 + 3.62 \ln(\dot{\epsilon}/0.0004) \\ + 0.01226(T - 81)\ln[\dot{\epsilon}/(2.053 \times 10^{18})] \quad (7)$$

The second term on the right hand side of eq. (7) reflects the offset in σ_{Yo} at T_g , whereas the last term contains the linear dependence of σ_{Yo} on T,

both because of testing at a strain rate $\dot{\varepsilon}$. The predictions of eq. (7) were compared with reasonable success with experimental data for the same PVC material at T = 32 and 75°C, as shown in the lines on Figure 5. Further comparisons are shown in Figure 7, where the yield loci, which correspond to eq. (7), are in satisfactory agreement with experimental points at temperatures of 32 and 75°C.

CONCLUSIONS

A full program of testing tubular specimens of commercial PVC pipe material at all stress combinations of the plane-stress space indicated that yielding closely follows the von Mises yield criterion at various strain rates and temperatures. This behavior was confirmed for such PVC compounds, which possess the same tensile and compressive yield strength, because yielding occurred at a constant volume independently of hydrostatic stress and a strain vector normal to the yield locus. The large deviations from the von Mises criterion existing for subsequent yielding loci may be explained phenomenologically by craze formation. The viscous yield behavior of the tested PVC material was represented by an Eyring-Wardtype working expression relating yield stress to temperature and strain rate.

NOMENCLATURE

ΔE^*	activation energy
T	temperature
$T_{ m g}$	glass-transition temperature
R°	gas constant
υ	activation volume for shear flow
Ω	pressure activation volume
ε	strain
ε	effective strain
έ	strain rate
$\dot{\overline{\epsilon}}$	effective strain rate
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
$\bar{\sigma}_Y$	effective yield stress
σ_m	mean or hydrostatic stress
σ_{YT}	tensile yield stress
σ_{YC}	compressive yield stress
au	maximum shear stress
$ au^{ m oct}$	octahedral shear stress
$ au_c$	shear yield stress at no pressure
α	coefficient of friction in a modified
	Tresca yield criterion

$ au_o$	octahedral shear yield stress at no
	pressure
μ	coefficient of pressure in a modified
	von Mises yield criterion
PVC	poly(vinyl chloride)
PMMA	poly(methyl methacrylate)
PC	polycarbonate
\mathbf{PS}	polystyrene
HDPE	high-density polyethylene

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